LETTER

Ring-on-ring strength measurements on rectangular glass slides

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The ring-on-ring test [1–6] has been standardized and used extensively for strength measurements on glasses and ceramics. Whereas bend bars suffer from flaws at their edges that dominate the strength, the edges of carefully prepared circular disks for ring-on-ring tests almost never fail from their edges. For these specimens, the mechanical defects in the surface determine their strength.

Very often, specimens are of interest which are not of a circular shape and yet are tested using the ring-on-ring configuration. The stress and strain distributions of these specimens can deviate strongly from that of the circular disk for which stress–strain relations are available from literature [1]. This is the case for glass slides of rectangular shape. Figure 1a shows a slide of 25×75 mm in dimension, which is loaded by two rings 11 mm and 22 mm in diameter.

In order to determine the stress distributions of such specimens, finite element computations were carried out with 1 mm and 1.5 mm slide thickness. The resulting tangential stresses, σ_t, and radial stresses, σ_r, on the tensile surface are given in a normalised representation

$$\sigma_{t,r} = \frac{3F}{4\pi t^2} D_{t,r}(v) \tag{1}$$

where F is the applied load, v is Poisson's ratio, and t is the plate thickness.

The results in terms of $D_{t,r}$ are plotted in Fig. 2a along the x-axis, and in Fig. 2b along the y-axis. The stresses are almost constant in the range of $x, y < r_1$ (for r_1 see Fig. 1b). The strong local stress peaks at the outer ring are caused by the concentrated contact effects between ring and specimen.

Figure 3a represents the biaxiality ratio along the two symmetry axes. This ratio is found to be very close to $\sigma_r/\sigma_t = 1$, with maximum deviations of about 1%, i.e. with sufficient accuracy. The test exhibits an equibiaxial stress state within the inner ring. The influence of Poisson's ratio is illustrated in Fig. 3b.

In the range of $0.2 \leq v \leq 0.25$ (relevant for glass), the coefficients D_t and D_n (at $x = y = 0$) for the specially chosen geometry can be approximated as

$$D_t \cong 1.68 + 1.12v \tag{2}$$

$$D_r \cong 1.666 + 1.15v \tag{3}$$

In an earlier paper [7], strength data of rectangular slides were computed with the well-known disk formula. For a disk-shaped test specimen the equibiaxial stresses in the inner ring, expressed by Eq. (1), are [1, 2]

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Fig. 1 Ring-on-ring test for (**a**) rectangular glass slides (microscope slides), (**b**) standard test on disks
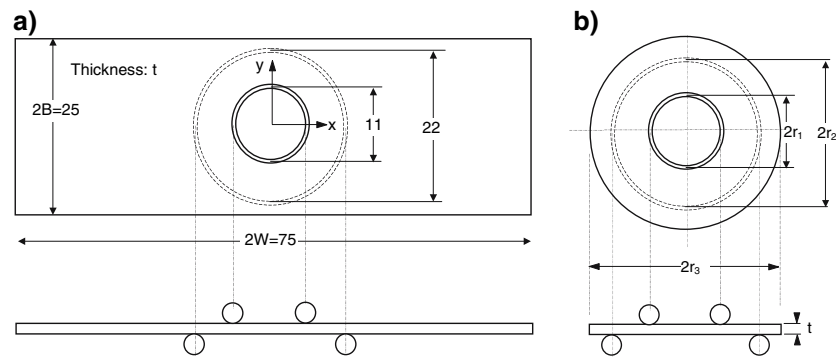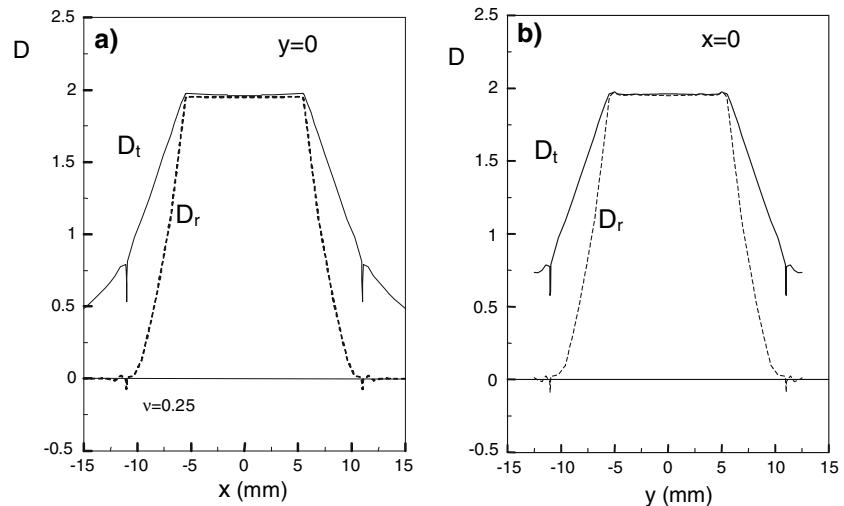

Fig. 2 Normalised tangential and radial stresses (**a**) along the x-axis, (**b**) along the y-axis (for $v = 0.25$)

$$D_r = D_t = (1 - v)\frac{1 - (r_1/r_2)^2}{(r_3/r_2)^2} - 2(1 + v)\log(r_1/r_2) \quad (4)$$

(for geometric data see Fig. 1b). By comparing Eq. (4) with the finite element results of Fig. 2, an appropriate effective value of r_3/r_2 can be determined for the rectangular specimen. It results

$$r_{3,\text{eff}} = 1.57r_2 \cong 1.38B \quad (5)$$

A commercial soda-lime glass with a high content of alkali and alkaline earth oxides was investigated in [7] (AR glass, Schott GmbH, Mainz)[1]. It consists (in wt%) of 69% SiO_2, 13% Na_2O, 5% CaO, 4% Al_2O_3, 3% MgO, 3% K_2O, 2% BaO, and 1% B_2O_3 and has a Poisson's ratio of $v = 0.22$.

Specimens of 25 mm $\times$ 75 mm $\times$ 1 mm with their 25 mm $\times$ 75 mm faces polished were annealed for 5 h at 430 °C. Then, 50 specimens were stored in water for

250 h at room temperature, while a second series of 48 specimens was stored for 230 h at 90 °C. The specimens were cooled to room temperature and water was removed by drying the glass specimens with soft facial tissues. Only the polished surfaces were tested. The strength data are plotted in Fig. 4 in a Weibull representation. The Weibull parameters and 90% confidence intervals were determined according to [8, 9] (see Table 1.). Since the 90% intervals do not overlap, the difference in strength is significant.

Possible explanations of the strength differences were discussed in [7]. Stress effects due to the generation of ion exchange layers and also crack healing effects were taken into account. Surface hydration consists of the interdiffusion of either hydrogen ions (H^+) or hydronium ions (H_3O^+) with the Na^+ ions in the glass. The H^+/Na^+ exchange results in a tensile stress in the hydration layer, because the H^+ ion is smaller than the Na^+ ion. In contrast to this, H_3O^+/Na^+ exchange leads to a compressive stress, because H_3O^+ is larger than Na^+. These stresses must affect the strength.

From the increased strengths of specimens stored at 90 °C, it can be concluded that compressive stresses

[1] The use of commercial names is only for purposes of identification and does not imply endorsement by the National Institute of Standards and Technology.

Fig. 3 (**a**) Ratio of the radial and tangential stresses, (**b**) influence of Poisson's ratio on coefficients D_t, D_r
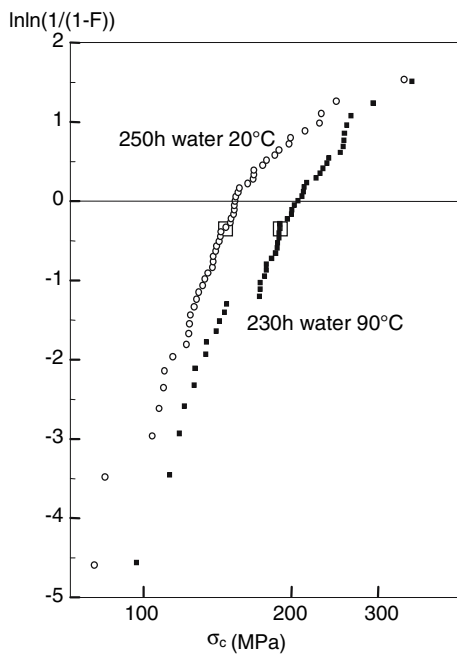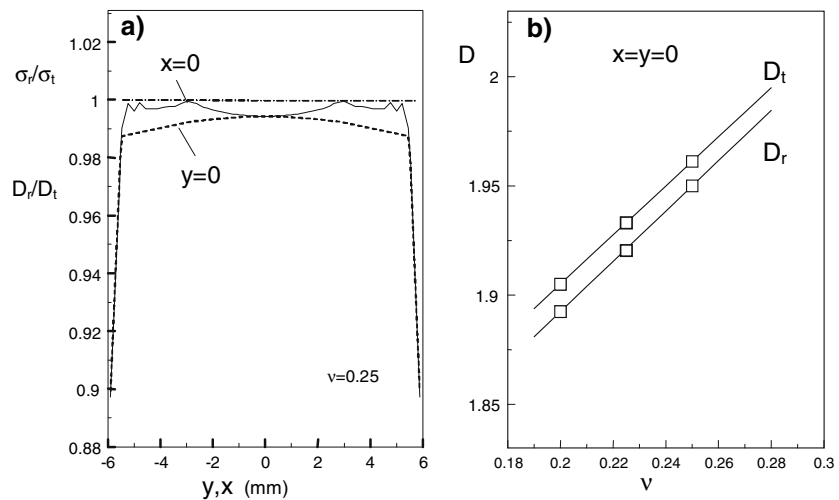

Fig. 4 Strength data obtained from ring-on-ring tests in a Weibull representation, squares: median values

must be generated during ''high-temperature'' water storage. In [7] the strengths were computed under the assumption of $r_{3, \text{eff}} = B$. This resulted in a compressive stress in the ion exchange layer of –2.4 GPa. From the evaluation of the finite element results presented before, a slightly reduced compressive stress of –2.2 GPa can now be concluded.

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Table 1 Weibull parameters and 90% confidence intervals (data in brackets) of strength

	σ_0 (MPa)	m	m_{corr}
Water 20 °C	167.9 [156.0; 180.7]	3.43 [2.8; 4.0]	3.33
Water 90 °C	209.6 [196.8; 223.3]	4.07 [3.3; 4.8]	3.95